

Nonfactorizable contributions to weak $D \rightarrow PV$ decays

K. K. Sharma[†], A. C. Katoch[†] and R. C. Verma¹

International Centre of Theoretical Physics, Trieste, Italy.

[†]Centre of Advanced Study in Physics, Department of Physics

Panjab University, Chandigarh-160 014, India.

ABSTRACT

We investigate nonfactorizable contributions to two-body hadronic decays of the charmed mesons to a pseudoscalar meson and a vector meson in Cabibbo-favored mode. Employing SU(3)-flavor symmetry for the nonfactorizable matrix elements, we obtain branching ratios of the decays in consistent agreement with experiment.

PACS numbers: 13.25. m, 14.40. Cs, 14.40.Jz.

¹Permanent Address: Centre of Advanced Study in Physics, Department of Physics, Panjab University, Chandigarh-160 014, India.

I. Introduction

With the availability of extensive data on two-body weak hadronic decays of heavy flavor mesons [1], it has now become possible to test the validity of the factorization model which has been considered to be supported by the D meson phenomenology [2-4]. In the recent works [5-8], it has been shown that the factorization model fails to account for the observed data on charmed and bottom meson hadrons. Large N_c limit, in which nonfactorizable contributions are usually ignored, does not work when extended to B meson decays as these clearly demand [6] a positive value of the QCD parameter a_2 . Even in the D meson decays, universal choice of the parameters a_1 and a_2 does not explain many of the hadronic decays of D and D_s mesons. For instance, the measured branching ratios of η and η' emitting Cabibbo-angle-favored decays of charmed mesons are considerably larger than those predicted in the spectator quark picture. Annihilation terms, if used to bridge the discrepancy between theory and experiment, require large form factors particularly for $D^0 \rightarrow \eta/\eta' + \bar{K}^0$ and $D^0 \rightarrow \eta + \bar{K}^{*0}$ decays [7]. Further, factorization also fails to relate $D_s^+ \rightarrow \eta/\eta' + \pi^+/\rho^+$ decays with semileptonic decays $D_s^+ \rightarrow \eta/\eta' + e^+ + \nu$ [7, 8] in a consistent manner.

Recently, there has been a growing interest in studying the nonfactorizable terms for weak hadronic decays of the heavy flavor hadrons. Many attempts have been made to estimate the amount of nonfactorizable effects needed to reproduce the experimental results for charmed and bottom sector [9-11] when real value $N_c = 3$ is used. In an earlier work [12], using isospin symmetry one of us (RCV) has searched for a systematics in these estimates for various decays of D^+ and D^0 mesons. It has been shown that the nonfactorizable isospin 1/2 and 3/2 reduced amplitudes may bear a universal ratio for $D \rightarrow$

$\bar{K}\pi/\bar{K}\rho/\bar{K}^*\pi/\bar{K}a_1/\bar{K}^*\rho$ decay modes. The formalism, when generalized to $SU(3)$ for studying $D \rightarrow PP$ decays, where P denotes a pseudoscalar meson, has resulted in a consistent fit with experiment [13].

In this work, we have extended the $SU(3)$ -flavor analysis of nonfactorizable contributions to $D \rightarrow PV$ decays, where V denotes a vector meson. We analyze the Cabibbo- favored decays of D^0 , D^+ and D_s^+ mesons taking into account the final state interactions (FSI). In section II, we develop the formalism. Results and discussion are given in the last section.

II. Formalism

We start with the effective weak Hamiltonian

$$H_w = \tilde{G}_F [c_1(\bar{u}d)(\bar{s}c) + c_2(\bar{s}d)(\bar{u}c)], \quad (1)$$

where $\tilde{G}_F = \frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*$ and $\bar{q}_1q_2(\equiv \bar{q}_1\gamma_\mu(1 - \gamma_5)q_2)$ represents color singlet V - A current and the QCD coefficients at the charm mass scale are

$$c_1 = 1.26 \pm 0.04, \quad c_2 = -0.51 \pm 0.05. \quad (2)$$

Due to the Fierz transformation of the product of two Dirac currents in (1) in N_c -color space, the Hamiltonian takes the following form [9]:

$$\begin{aligned} H_w^{CF} &= \tilde{G}_F [a_1(\bar{u}d)(\bar{s}c) + c_2 H_w^8], \\ H_w^{CS} &= \tilde{G}_F [a_2(\bar{s}d)(\bar{u}c) + c_1 \tilde{H}_w^8], \end{aligned} \quad (3)$$

for color favored (CF) and color suppressed (CS) decay amplitudes respectively. Here,

$$a_{1,2} = c_{1,2} + \frac{c_{2,1}}{N_c}, \quad (4)$$

$$H_w^8 = \frac{1}{2} \sum_{a=1}^8 (\bar{u}\lambda^a d)(\bar{s}\lambda^a c),$$

$$\tilde{H}_w^8 = \frac{1}{2} \sum_{a=1}^8 (\bar{s} \lambda^a d) (\bar{u} \lambda^a c), \quad (5)$$

where $\bar{q}_1 \lambda^a q_2 (\equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) \lambda^a q_2)$ represents color octet current.

Matrix elements of the first terms in (3) can be calculated using the factorization scheme. These are given column (ii) of Table I. So long as one restricts to the color-singlet intermediate states, second terms in (3) are ignored and one usually treats a_1 and a_2 as input parameters in place of using $N_c = 3$ in reality. It is generally believed [2-4] that the $D \rightarrow \bar{K} \pi$ decays favor $N_c \rightarrow \infty$ limit, i.e.,

$$a_1 \approx 1.26, \quad a_2 \approx -0.51. \quad (6)$$

However, it has been shown that this does not explain all the decay modes of charm mesons [5,7]. For instance, the observed $D^0 \rightarrow \eta \bar{K}^0$ and $D^0 \rightarrow \eta' \bar{K}^0$ decay widths are larger than those predicted in the spectator quark model. Also in $D \rightarrow PV$ mode, measured branching ratios for $D^0 \rightarrow \bar{K}^0 \omega / \eta \bar{K}^{*0}$, $D^+ \rightarrow \bar{K}^{*0} \pi^+$, and $D_s^+ \rightarrow \bar{K}^0 K^{*+} / \eta \rho^+ / \eta' + \rho^+$, are considerably higher than those predicted by the spectator quark diagrams. In addition to the spectator quark diagram, factorizable W-exchange or W-annihilation diagrams may contribute to the weak nonleptonic decays. However, such contributions are normally expected to be suppressed [2] in the meson decays. For D^+ meson decays, these do not appear in the Cabibbo favored decay process. For D^0 meson decays, these are further color-suppressed as these involve lower QCD coefficient a_2 . Therefore, we have ignored them in the present analysis.

We now investigate nonfactorizable contributions to these decays. Matrix elements of H_w^8 and \tilde{H}_w^8 between charm mesons and two-body uncharmed final states are difficult to calculate theoretically [9,10], as these involve nonperturbative effects arising due to soft-gluon exchange. We employ SU(3)-flavor-

symmetry [14] to handle these matrix elements. In the SU(3) limit, the two Hamiltonians H_w^8 and \tilde{H}_w^8 behave like H_{13}^2 component of 6^* and 15 representations of the SU(3). Since H_w^8 and \tilde{H}_w^8 transform into each other under the interchange of u and s quarks, which forms V-spin subgroup of the flavor-SU(3), the reduced matrix elements satisfy

$$\langle PV || \tilde{H}_w^8 || D \rangle = \langle PV || H_w^8 || D \rangle. \quad (7)$$

The matrix elements $\langle PV | H_w^8 | D \rangle$, appearing in the nonfactorizable effects, are considered as *weak spurion* + *D meson* $\rightarrow P + V$ scattering process, whose general structure can be written as

$$\begin{aligned} \langle PV | H_w^8 | D \rangle = & [a_1(P_m^b V_a^m P^c) + a_2(P_a^m V_m^b P^c) \\ & + a_3(P_a^b V_m^c + P_m^b V_a^c) P^m] H_{[b,c]}^a \\ & + [b_1(P_m^b V_a^m P^c) + b_2(P_a^m V_m^b P^c) \\ & + b_3(P_a^b V_m^c P^m) + b_4(P_m^b V_a^c P^m)] H_{(b,c)}^a \\ & + [e_1(P_a^c V_m^m P^b) + e_2(P_m^m V_a^c P^b)] H_{[b,c]}^a \\ & + [d_1(P_a^c V_m^m P^b) + d_2(P_m^m V_a^c P^b)] H_{(b,c)}^a, \end{aligned} \quad (8)$$

where P^a denotes triplet of D-mesons $P^a \equiv (D^0, D^+, D_s^+)$ and P_b^a, V_b^a denote $3 \otimes 3$ matrices of uncharmed pseudoscalar meson and vector meson nonets respectively. For pseudoscalar mesons,

$$P_b^a = \begin{pmatrix} P_1^1 & \pi^+ & K^+ \\ \pi^- & P_2^2 & K^0 \\ K^- & \bar{K}^0 & P_3^3 \end{pmatrix} \quad (8)$$

with

$$P_1^1 = \frac{1}{\sqrt{2}}(\pi^0 + \eta \sin \theta + \eta' \cos \theta),$$

$$\begin{aligned}
P_2^2 &= \frac{1}{\sqrt{2}}(-\pi^0 + \eta \sin \theta + \eta' \cos \theta), \\
P_3^3 &= -\eta \cos \theta + \eta' \sin \theta,
\end{aligned} \tag{9}$$

where θ governs the $\eta - \eta'$ mixing, and is related to the physical mixing [1] as,

$$\theta = \theta_{ideal} - \phi_{Phy}. \tag{10}$$

For vector meson nonet,

$$V_b^a = \begin{pmatrix} V_1^1 & \rho^+ & K^{*+} \\ \rho^- & V_2^2 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & V_3^3 \end{pmatrix} \tag{11}$$

with

$$\begin{aligned}
V_1^1 &= \frac{1}{\sqrt{2}}(\rho^0 + \omega \cos \theta' + \phi \sin \theta'), \\
V_2^2 &= \frac{1}{\sqrt{2}}(-\rho^0 + \omega \cos \theta' + \phi \sin \theta'), \\
V_3^3 &= \omega \sin \theta' - \phi \cos \theta'.
\end{aligned}$$

For ideal $\omega - \phi$ mixing

$$\theta' = 0. \tag{12}$$

In addition to the nonfactorizable effects considered so far, there may also arise nonfactorizable effects involving product of color-singlet currents. However, these may be relatively suppressed [10]. Even if these are included here, it has been shown [13] that their contributions can be absorbed in the unknown reduced amplitudes appearing in (7) due to the similar structure in the SU(3) framework.

There exists a straight correspondence between the terms appearing in (7) and various quark level processes. The terms involving the coefficients a_1 , a_2 , b_1 and b_2 represent annihilation diagrams. Notice that, due to the involvement of gluons, these are no longer suppressed. The terms having

coefficients a_3 , b_3 and b_4 represent spectator-quark like diagrams where the uncharm quark in the parent D-meson flows into one of the final state mesons. The last terms having coefficients e'_i 's and d'_i 's are hair-pin diagrams, where $q\bar{q}$ pair generated in the process hadronizes to one of the final state mesons. Choosing H_{13}^2 component from (7), we obtain nonfactorizable contributions to various $D \rightarrow PV$ decays. These are given in column (ii) of Table II.

III. Results and Discussion

The decay rate formula for $D \rightarrow PV$ decays is given by

$$\Gamma(D \rightarrow PV) = |\tilde{G}_F|^2 \frac{p_c^3}{8\pi m_V^2} |A(D \rightarrow PV)|^2, \quad (13)$$

where p_c is the three-momentum of final state particles in the rest frame of D meson and m_V is the mass of vector meson emitted. Now we proceed to determine nonfactorizable effects to various decays. First, we determine the factorizable contributions to various decays using $N_c = 3$ which fixes,

$$a_1 = 1.09, \quad a_2 = -0.09, \quad (14)$$

ignoring the errors in the QCD coefficients c_1 and c_2 . For the form factors at zero momentum transfer, we use

$$\begin{aligned} F_1^{DK}(0) &= 0.76, \quad F_1^{D\pi}(0) = 0.83, \\ A_0^{DK^*}(0) &= 0.75, \end{aligned} \quad (15)$$

as guided by the semileptonic decays of D - mesons [1, 15-17], and

$$\begin{aligned} F_1^{D\eta}(0) &= 0.681, \quad F_1^{D\eta'}(0) = 0.655, \\ F_1^{Ds\eta}(0) &= 0.723, \quad F_1^{Ds\eta'}(0) = 0.704, \\ F_1^{DsK}(0) &= 0.760; \end{aligned} \quad (16)$$

$$\begin{aligned}
A_0^{D\rho}(0) &= 0.669, \quad A_0^{D\omega}(0) = 0.669, \\
A_0^{D\phi}(0) &= 0.669, \quad A_0^{D_s\omega}(0) = 0.700, \\
A_0^{D_s\phi}(0) &= 0.700, \quad A_0^{D_sK^*}(0) = 0.634,
\end{aligned} \tag{17}$$

from the BSW model [2] for want of experimental information. These form factors are then extrapolated in q^2 using a monopole form with appropriate pole masses. Following values are used for the meson decay constants [1,2] (in GeV)

$$\begin{aligned}
f_\pi &= 0.132, \quad f_K = 0.161, \\
f_\rho &= 0.212, \quad f_{K^*} = 0.221.
\end{aligned} \tag{18}$$

Numerical values of the factorizable amplitudes are given in col (iii) of Table I.

Notice that $D \rightarrow \bar{K}\rho$ and $D \rightarrow \bar{K}^*\pi$ decays involve elastic FSI whereas the remaining decays are not affected by them. As a result, the isospin amplitudes 1/2 and 3/2 appearing in these decays may develop different phases. We illustrate the procedure for $D \rightarrow \bar{K}\rho$ amplitudes:

$$\begin{aligned}
A(D^0 \rightarrow K^-\rho^+) &= \frac{1}{\sqrt{3}}[A_{3/2}e^{i\delta_{3/2}} + \sqrt{2}A_{1/2}e^{i\delta_{1/2}}], \\
A(D^0 \rightarrow \bar{K}^0\rho^0) &= \frac{1}{\sqrt{3}}[\sqrt{2}A_{3/2}e^{i\delta_{3/2}} - A_{1/2}e^{i\delta_{1/2}}], \\
A(D^+ \rightarrow \bar{K}^0\rho^+) &= \sqrt{3}A_{3/2}e^{i\delta_{3/2}}.
\end{aligned} \tag{19}$$

Following phase independent relations:

$$\begin{aligned}
|A(D^0 \rightarrow K^-\rho^+)|^2 + |A(D^0 \rightarrow \bar{K}^0\rho^0)|^2 &= |A_{1/2}|^2 + |A_{3/2}|^2, \\
|A(D^+ \rightarrow \bar{K}^0\rho^+)|^2 &= 3|A_{3/2}|^2,
\end{aligned} \tag{20}$$

allow us to work without the phases. Writing the total decay amplitude as a sum of the factorizable and nonfactorizable parts

$$A(D \rightarrow PV) = A^f(D \rightarrow PV) + A^{nf}(D \rightarrow PV), \tag{21}$$

we obtain

$$A_{1/2}^{nf}(D \rightarrow \bar{K}\rho) = \frac{1}{\sqrt{3}}\{\sqrt{2}A^{nf}(D^0 \rightarrow K^-\rho^+) - A^{nf}(D^0 \rightarrow \bar{K}^0\rho^0)\}, \quad (22)$$

$$\begin{aligned} A_{3/2}^{nf}(D \rightarrow \bar{K}\rho) &= \frac{1}{\sqrt{3}}\{A^{nf}(D^0 \rightarrow K^-\rho^+) + \sqrt{2}A^{nf}(D^0 \rightarrow \bar{K}^0\rho^0)\} \\ &= \frac{1}{\sqrt{3}}A^{nf}(D^+ \rightarrow \bar{K}^0\rho^+). \end{aligned} \quad (23)$$

The last relation (23) leads to the following constraint:

$$\frac{A_{1/2}^{nf}(D \rightarrow \bar{K}\rho)}{A_{3/2}^{nf}(D \rightarrow \bar{K}\rho)} = \frac{c_1^2 + 2c_2^2}{\sqrt{2}(c_2^2 - c_1^2)} = -1.123 \pm 0.112. \quad (24)$$

Experimental value $B(D^+ \rightarrow \bar{K}^0\rho^+) = 6.6\%$ then predicts sum of the branching ratios of $D^0 \rightarrow \bar{K}\rho$ modes:

$$B(D^0 \rightarrow K^-\rho^+) + B(D^0 \rightarrow \bar{K}^0\rho^0) = 10.42\%, \quad (11.50 \pm 1.31\% \text{ Expt.}) \quad (25)$$

in good agreement with experiment. Present data [1] on these modes is consistent with a choice of zero phase difference between isospin 1/2 and 3/2 channels and fixes the ratio of nonfactorizable amplitudes,

$$\frac{A_{1/2}^{nf}(D \rightarrow \bar{K}\rho)}{A_{3/2}^{nf}(D \rightarrow \bar{K}\rho)} = -1.481 \pm 0.582 \text{ (Expt.)} \quad (26)$$

consistent with theoretical value given in Eq. (24). For branching ratio of the individual modes, we obtain

$$B(D^0 \rightarrow K^-\rho^+) = 9.37\% \quad (10.4 \pm 1.3\% \text{ Expt.}), \quad (27)$$

$$B(D^0 \rightarrow \bar{K}^0\rho^0) = 1.05\%, \quad (1.10 \pm 0.18\% \text{ Expt.}). \quad (28)$$

Performing a similar analysis for the $D \rightarrow \bar{K}^*\pi$ decay amplitudes and using the experimental value $B(D^+ \rightarrow \bar{K}^{*0}\pi^+) = 2.2\%$, we calculate:

$$B(D^0 \rightarrow K^{*-}\pi^+) + B(D^0 \rightarrow \bar{K}^{*0}\pi^0) = 7.44\%, \quad (7.9 \pm 0.7\%, \text{ Expt.}) \quad (29)$$

in nice agreement with experiment. For these modes also, the isospin reduced amplitudes bear the same ratio as given in Eq.(24),

$$\frac{A_{1/2}^{nf}(D \rightarrow \pi \bar{K}^*)}{A_{3/2}^{nf}(D \rightarrow \pi \bar{K}^*)} = \frac{c_1^2 + 2c_2^2}{\sqrt{2}(c_2^2 - c_1^2)} = -1.123 \pm 0.112, \quad (30)$$

which compares well with experimental value -1.171 ± 0.158 when negative sign is chosen for $A_{3/2}$.

Calculation of branching ratio of the remaining $D \rightarrow PV$ decays needs numerical values of the reduced amplitudes. Apparently these decays seem to involve several unknown parameters. However, the parameters d 's and e 's appear only when isosinglet meson is emitted. Also that D meson decays involve combinations $(a_1 - b_1)$ and $(a_2 - b_2)$ which in fact are expressible as

$$\begin{aligned} a_1 - b_1 &= [-(c_1 + c_2)a_3 + c_2b_3 + c_1b_4]/(c_1 - c_2), \\ a_2 - b_2 &= [(c_1 + c_2)a_3 + c_1b_3 + c_2b_4]/(c_1 - c_2), \end{aligned} \quad (31)$$

where

$$a_3 = -0.042 \text{ GeV}^2, \quad (32)$$

$$b_3 + b_4 = -0.251 \text{ GeV}^2, \quad (33)$$

are given by D^+ modes. The relations (31) follow from the constraints given in Eqs. (24) and (30). With the experimental values $B(D^0 \rightarrow \bar{K}^0 \phi) = 0.83\%$, and $B(D^0 \rightarrow \bar{K}^0 \omega) = 2.0\%$, and taking negative and positive signs for their experimental amplitudes, we find (in GeV^2),

$$b_3 = 0.042, \quad (34)$$

$$b_4 = -0.293, \quad (35)$$

$$e_1 + d_1 = -0.047. \quad (36)$$

The relations given in (31) then yield:

$$a_1 - b_1 = -0.202, \quad (37)$$

$$a_2 - b_2 = 0.108. \quad (38)$$

Using the measured branching ratios $B(D_s^+ \rightarrow K^+ \bar{K}^{*0}) = 3.3\%$ and $B(D_s^+ \rightarrow \bar{K}^0 K^{*+}) = 4.2\%$, we find (in GeV^2),

$$a_1 + b_1 = 0.190, \quad (39)$$

$$a_2 + b_2 = 0.123, \quad (40)$$

by taking negative and positive signs of their experimental amplitudes respectively. These parameters then predict

$$B(D_s^+ \rightarrow \pi^0 \rho^+) = B(D_s^+ \rightarrow \pi^+ \rho^0) = 0.10\% \quad (41)$$

which is well below the experimental upper limit ($< 0.28\%$) for $B(D_s^+ \rightarrow \pi^+ \rho^0)$. Further, experimental value $B(D \rightarrow \pi^+ \phi) = 3.5\%$ yields,

$$e_1 - d_1 = 0.070 \text{ GeV}^2, \quad (42)$$

for the positive choice of its experimental amplitude. This value in turn leads to:

$$B(D \rightarrow \pi^+ \omega) = 0.61\% \quad (43)$$

obeying the experimental upper limit ($< 1.7\%$). Now, we are left with η and η' emitting decays which involve mixing angle ϕ_{Phy} . We have chosen to present results for all the three mixing angles -10° , -19° , and -23° given in the Particle Data Group [1] so as to make the trend with mixing angle evident. Measured branching ratios $B(D^0 \rightarrow \eta \bar{K}^{*0}) = 1.9\%$ and $B(D_s^+ \rightarrow \eta' \rho^+) = 12.0\%$ fix the parameters:

$$e_2 + d_2 = 0.668 \text{ GeV}^2 \text{ for } \phi_{Phy} = -10^\circ$$

$$\begin{aligned}
&= 0.373 \text{ GeV}^2 \text{ for } \phi_{Phy} = -19^0, \\
&= 0.313 \text{ GeV}^2 \text{ for } \phi_{Phy} = -23^0,
\end{aligned} \tag{44}$$

and

$$\begin{aligned}
e_2 - d_2 &= 0.457 \text{ GeV}^2 \text{ for } \phi_{Phy} = -10^0 \\
&= 0.428 \text{ GeV}^2 \text{ for } \phi_{Phy} = -19^0, \\
&= 0.412 \text{ GeV}^2 \text{ for } \phi_{Phy} = -23^0,
\end{aligned} \tag{45}$$

for negative and positive signs of respective experimental amplitudes. Calculated branching ratios for η and η' emitting decays are listed in columns (ii) to (iv) of Table III for different values of the mixing angles. For the sake of comparison with factorizable part, nonfactorizable contributions to various decays are given in column (iii) of Table II.

Acknowledgments

One of the authors (RCV) is grateful to Professor A. Salam, the President, and Professor G. Virasoro, the Director, ICTP, Trieste, UNESCO, and Dr. S. Randjbar-Daemi for the hospitality at the ICTP, Trieste.

Table I

Spectator-quark decay amplitudes

Process	Amplitude (A^f)	$a_1 = 1.09, a_2 = -0.09$
$D^0 \rightarrow K^- \rho^+$	$2a_1 f_\rho m_\rho F_1^{DK}(m_\rho^2)$	0.316
$D^0 \rightarrow \bar{K}^0 \rho^0$	$\sqrt{2}a_2 f_K m_\rho A_0^{D\rho}(m_K^2)$	-0.011
$D^0 \rightarrow \bar{K}^0 \omega$	$\sqrt{2}a_2 f_K \cos\theta' m_\omega A_0^{D\omega}(m_K^2)$	-0.011
$D^0 \rightarrow \bar{K}^0 \phi$	$-\sqrt{2}a_2 \sin\theta' f_K m_\phi A_0^{D\phi}(m_K^2)$	0
$D^0 \rightarrow \pi^+ K^{*-}$	$2a_1 f_\pi m_{K^*} A_0^{DK^*}(m_\pi^2)$	0.179
$D^0 \rightarrow \pi^0 \bar{K}^{*0}$	$\sqrt{2}a_2 f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)$	-0.026
$D^0 \rightarrow \eta \bar{K}^{*0}$	$\sqrt{2}a_2 f_{K^*} \sin\theta m_{K^*} F_1^{D\eta}(m_{K^*}^2)$	-0.015
$D^0 \rightarrow \eta' \bar{K}^{*0}$	$\sqrt{2}a_2 f_{K^*} \cos\theta m_{K^*} F_1^{D\eta'}(m_{K^*}^2)$	-0.015
$D^+ \rightarrow \bar{K}^0 \rho^+$	$2a_1 f_\rho m_\rho F_1^{DK}(m_\rho^2)$	0.300
$D^+ \rightarrow \pi^+ \bar{K}^{*0}$	$+2a_2 f_K m_\rho A_0^{D\rho}(m_K^2)$	
	$2a_1 f_\pi m_{K^*} A_0^{DK^*}(m_\pi^2)$	0.143
	$+2a_2 f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)$	
$D_s^+ \rightarrow \pi^+ \rho^0$	0	0
$D_s^+ \rightarrow \pi^0 \rho^+$	0	0
$D_s^+ \rightarrow \pi^+ \omega$	$2a_1 f_\pi \sin\theta' m_\omega A_0^{D_s\omega}(m_\pi^2)$	0
$D_s^+ \rightarrow \pi^+ \phi$	$2a_1 f_\pi \cos\theta' m_\phi A_0^{D_s\phi}(m_\pi^2)$	0.204
$D_s^+ \rightarrow \eta \rho^+$	$-2a_1 f_\rho \cos\theta m_\rho F_1^{D_s\eta}(m_\rho^2)$	-0.209
$D_s^+ \rightarrow \eta' \rho^+$	$2a_1 f_\rho \sin\theta m_\rho F_1^{D_s\eta'}(m_\rho^2)$	0.205
$D_s^+ \rightarrow K^+ \bar{K}^{*0}$	$2a_2 f_{K^*} m_{K^*} F_1^{D_s K}(m_{K^*}^2)$	-0.033
$D_s^+ \rightarrow \bar{K}^0 K^{*+}$	$2a_2 f_K m_{K^*} A_0^{D_s K^*}(m_K^2)$	-0.018

Table II

Nonfactorizable contributions to $D \rightarrow PV$ decays. $(c_1$ and c_2 are the QCD coefficients)

Process	Nonfactorizable contribution	$\phi = -10^\circ$
$D^0 \rightarrow K^- \rho^+$	$c_2[-a_1 - a_3 + b_1 + b_4]$	0.024
$D^0 \rightarrow \bar{K}^0 \rho^0$	$\frac{1}{\sqrt{2}}c_1[a_1 - a_3 - b_1 + b_3]$	-0.103
$D^0 \rightarrow \bar{K}^0 \omega$	$c_1[\frac{\cos\theta'}{\sqrt{2}}(-a_1 - a_3 + b_1 + b_3 + 2(e_1 + d_1)) + \sin\theta'(-a_2 + b_2 + e_1 + d_1)]$	0.174
$D^0 \rightarrow \bar{K}^0 \phi$	$c_1[\frac{\sin\theta'}{\sqrt{2}}(-a_1 - a_3 + b_1 + b_3 + 2(e_1 + d_1)) - \cos\theta'(a_2 - b_2 - e_1 - d_1)]$	-0.199
$D^0 \rightarrow \pi^+ K^{*-}$	$c_2[-a_2 + a_3 + b_2 + b_3]$	0.058
$D^0 \rightarrow \pi^0 \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}c_1[a_2 + a_3 - b_2 + b_4]$	-0.201
$D^0 \rightarrow \eta \bar{K}^{*0}$	$c_1[\frac{\sin\theta}{\sqrt{2}}(-a_2 + a_3 + b_2 + b_4 + 2(e_2 + d_2)) + \cos\theta(a_1 - b_1 - e_2 - d_2)]$	-0.210
$D^0 \rightarrow \eta' \bar{K}^{*0}$	$c_1[\frac{\cos\theta}{\sqrt{2}}(-a_2 + a_3 + b_2 + b_4 + 2(e_2 + d_2)) + \sin\theta(-a_1 + b_1 + e_2 + d_2)]$	1.334
$D^+ \rightarrow \bar{K}^0 \rho^+$	$(c_1 + c_2)[-2a_3 + b_3 + b_4]$	-0.122
$D^+ \rightarrow \pi^+ \bar{K}^{*0}$	$(c_1 + c_2)[2a_3 + b_3 + b_4]$	-0.254
$D_s^+ \rightarrow \pi^+ \rho^0$	$\frac{1}{\sqrt{2}}c_2[-a_1 + a_2 - b_1 + b_2]$	0.024
$D_s^+ \rightarrow \pi^0 \rho^+$	$\frac{1}{\sqrt{2}}c_2[a_1 - a_2 + b_1 - b_2]$	0.024
$D_s^+ \rightarrow \pi^+ \omega$	$c_2[\frac{\cos\theta'}{\sqrt{2}}(a_1 + a_2 + b_1 + b_2 - 2(e_1 - d_1)) + \sin\theta'(a_3 + b_3 - e_1 + d_1)]$	-0.062
$D_s^+ \rightarrow \pi^+ \phi$	$c_2[\frac{\sin\theta'}{\sqrt{2}}(a_1 + a_2 + b_1 + b_2 - 2(e_1 - d_1)) - \cos\theta'(-a_3 - b_3 + e_1 - d_1)]$	0.037
$D_s^+ \rightarrow \eta \rho^+$	$c_2[\frac{\sin\theta}{\sqrt{2}}(a_1 + a_2 + b_1 + b_2 - 2(e_2 - d_2)) + \cos\theta(a_3 - b_4 + e_2 - d_2)]$	-0.097
$D_s^+ \rightarrow \eta' \rho^+$	$c_2[\frac{\cos\theta}{\sqrt{2}}(a_1 + a_2 + b_1 + b_2 - 2(e_2 - d_2)) + \sin\theta(-a_3 + b_4 - e_2 + d_2)]$	0.424
$D_s^+ \rightarrow K^+ \bar{K}^{*0}$	$c_1[a_1 + a_3 + b_1 + b_4]$	-0.186
$D_s^+ \rightarrow \bar{K}^0 K^{*+}$	$c_1[a_2 - a_3 + b_2 + b_3]$	0.263

Table III
Branching ratios (%) of η/η' emitting decays for different mixing angles.

Decay	$\phi_{phy} = -10^\circ$	$\phi_{phy} = -19^\circ$	$\phi_{phy} = -23^\circ$	Experiment
$D^0 \rightarrow \eta \bar{K}^{*0}$	1.90 ^a	1.90 ^a	1.90 ^a	1.9 ± 0.4
$D^0 \rightarrow \eta' \bar{K}^{*0}$	0.33	0.10	0.08	< 0.11
$D_s^+ \rightarrow \eta \rho^+$	10.47	5.23	3.50	10.0 ± 2.2
$D_s^+ \rightarrow \eta' \rho^+$	12.00 ^a	12.00 ^a	12.00 ^a	12.0 ± 3.0

^a input

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